Chapter -1

NUMBER SYSTEM

THE ULTIMATE FAMILY TREE

REAL NUMBERS

RATIONAL NUMBER
any number that can be

Expressed in $\frac{p}{q}$ form

Where q $\neq$ 0, p & q are co-prime.

(Further has family of 3)

NATURAL NUMBERS:-
Numbers that begin with 1,2,3……..etc.
WHOLE NUMBER

Numbers that being with 0,1, 2, 3,------etc.

INTEGERS

Set of numbers, consist of natural numbers, negative of natural numbers.

IRRATIONAL NUMBER

Any number that cannot be expressed in p/q form.

IMPORTANT SYMBOLS/ ABBREVATIONS
N:- the set of natural numbers.
W:- set of whole numbers
Z:- set of integers.
Q:- set of rational number .
R:- set of real number.
N → W → Z → Q → R
N C W C Z C Y C R

SOME IMPORTANT TERMS AND IMPORTANT POINTS

∗The set N of natural numbers is infinite, i. e it has unlimited members.

∗ N has smallest element namely '1'.

∗ N has no largest element i.e give me any natural number, we can find the bigger no from the given number.

∗ '0' is not the number of set N

∗ Set of whole numbers is infinite

∗ Smallest whole no is 'o'

∗ Set of whole numbers has no largest member

∗ Every natural no is a whole no.

∗ Non- zero smallest whole no is 1.

∗ The set Z of integers has neither the greatest number the least element.

∗ Every natural number is an integer

∗ Every whole number is an integer

CONSECUTIVE NUMBER: - A series of natural no each differing by one is called consecutive number eg So, S1, S2 etc.

PRIME NUMBER: - Number that has only two factors namely one and it self eg:- 2,3,5,7,11,13 etc.

→ 1 is not a prime number

→ Only even prime number is 2 all are odd.

→ Smallest prime number is 2.

TWIN PRIME: - Pair of prime numbers is said to be twin prime if they differ by for eg:- (3,5), (11,13), (17, 19), (29,31), (41, 43), (71, 73) are all twin- prime.

COMPOSITE NUMBERS: - those numbers which can be expressed as product of primes, 4, 6, 12 etc.

→ 1 is neither prime, nor composite

→ 1 is not composite.

→ 4 is the smallest composite number.

CO- PRIME NUMBERS:- A pair of no is said to be co-prime if the numbers have no- common factor other than one for eg:- 29 and 31 are co-prime.

PERFECT NUMBER:- A number is said to be perfect if it is equal to sum of its factors ones than itself for eg:-

6 = (1 + 2 + 3)

28 = (1 + 2 + 4 + 7 + 14)

6 and 8 are perfect numbers.

CLASSIFICATION OF RATIONAL & IRRATIONAL

1. TERMINATING DECIMALS:- rational numbers with a finite decimal part after finite numbers of steps are known as terminating decimals. Eg:- $\frac{1}{2}$ = 0.5, $\frac{7}{8}$ = 0 - 875etc.

2. NON TERMINATING DECIMALS:- those numbers in which the division process never comes to an end.

→ Repeating securing block of digits repeat itself eg:- 0.333-- $\left(\frac{1}{3}\right)$

$\frac{2}{3}$ = 0.6666---

0.1717----

0. 186186----

Non repeating $\frac{22}{7}$ =3.14159265 $\sqrt{2}$ =1.414 -- $\sqrt{3}$ = 1.732--

Pure recurring decimals – a decimals in which all the digits after the decimal point are repeated eg :- 0.66= 0.$\sqrt{6}$,0.$\sqrt{23}$

Mixed recurring decimals:- A decimal in which at least- one 0f the digits after decimal repeats eg:- 0.16

→ Every integer is a rational number

→ Every terminating decimal is a rational no.

→ Every recurring decimal is a rational number

→ A non-terminating repeating decimal is called recurring decimal.

→ Between any two rational numbers there are infinite number of rational numbers.

→ Every rational numbers can be represented in the form of terminating or non- terminating recurring decimal.

→ A number is irrational no it has a non- terminating and non- repeating decimal representation.

REAL NUMBERS:- rational numbers and irrational number taken together form the set of real numbers.

Denoted by R, $\sqrt{3}$, Z-5 etc.

NOTE:- π is defined as ratio of circumference of a circle to the length of the diameter.π $\sqrt{3}$ an irrational number since value of π is π = 3.14159265—which is neither terminating non repeating

IMPORTANT PROPERTIES OF IRRATIONAL NUMBERS

PROPERTY-1:- negative of an irrational number is an irrational number may be rational or irrational

PROPERTY- 2:- Sum of rational and irrational number is irrational number

PROPERTY- 3:- Product of non- zero rational and irrational number is irrational.

EXCEPTION:- O is rational number $\sqrt{3}$ is irrational
Ox $\sqrt{3}$ =0 Which is rational

PROPERTY-4:- Division of non-zero rational number and an irrational number is irrational

EXCEPTION:- 0 is rational number $\sqrt{3}$ is irrational number $\frac{0}{\sqrt{3}}$ = 0 which is rational number

PROPERTY-5:- sum of two irrational number may be rational or irrational.eg $\sqrt{2}$ + $\sqrt{3}$ is irrational but (2 + $\sqrt{3}$) + $(2 - \sqrt{3})$ = 4 is rational

PROPERTY-6:- Difference of two irrational number may be rational or irrational

Eg:- $\sqrt{2}$ + 2, 2 - $\sqrt{2}$ = 2$\sqrt{2}$ is irrational but 2 + $\sqrt{2}$, -2 + $\sqrt{2}$ = 4 is rational

PROPERTY-7:- product of two irrational number may be rational or irrational.

Eg:- $\sqrt{2}$ x $\sqrt{3}$ = $\sqrt{6}$ is an irrational but ($\sqrt{3}$ – 2) ($\sqrt{3}$ + 2) = $(\sqrt{3})^2 - (2)^2$ = 3 -4 = 1 is rational number.

PROPERTY-8:- division of two irrational number may be rational or irrational .

Eg:- $\sqrt{18} \div \sqrt{3}$ = $\sqrt{6}$ is irrational, $\sqrt{125} \div \sqrt{5}$ = 5 is rational

IMPORTANT POINTS

→ A rational number is either terminating or non- terminating but repeating q can be put in p/q form.

SOME USEFUL LAWS

LAWS-1 :- if a & b are positive real numbers then $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Eg:- $\sqrt{5x3} = \sqrt{5} \times \sqrt{3}$ =etc.

LAWS-2:- If a and b are two real numbers then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Eg:- $\sqrt{\frac{7}{5}} = \frac{\sqrt{7}}{\sqrt{5}}$ etc.

LAWS-3:- If a and b are positive real numbers then $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$= a-b

$(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = (\sqrt{5})^2 - (\sqrt{3})^2$ = 5-3=2

LAWS-4:- If a and b are positive real numbers ten $(a+ \sqrt{b})(a- \sqrt{b}) = a^2$-b

Eg:- $(6 + \sqrt{2})(6 - \sqrt{2}) = (6)^2 - (\sqrt{2})^2$ = 36-2 =34

LAWS-5:- if a and b are two real numbers, $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

Eg:- $(\sqrt{5} + \sqrt{3})(\sqrt{6} + \sqrt{7}) = \sqrt{5 \times 6} + \sqrt{5 \times 7} + \sqrt{3 \times 6} + \sqrt{3 \times 7}$

LAWS-6:- If a and b are positive real numbers then $(\sqrt{a} + \sqrt{b})^2$= a+ 2 $\sqrt{ab}$ + b.

LAWS OF EXPONENTS

LAW-1

$a^m \times a^n = a^{m+n}$

LAW-2

$\frac{a^m}{a^n} = a^{m-n}$

LAW-3

$(a^m)^n = a^{mn} = (a^n)^m$

LAW-4

$(ab)^n = a^n b^n$

LAW-5

$(a/b)^n = \frac{a^n}{b^n}$, $b \neq 0$

LAW-6

$a^o = 1$

LAW-7

$a^m \times b^m = (ab)^m$

LAW-8

$a^{-m} = \frac{1}{a^m}$

LAW-9

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

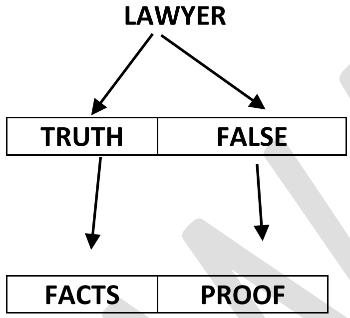
LAW-10

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

NOTE:- ($\sqrt{a}$) = (a) $^{1/2}$ (i.e square root means power half)
$\sqrt[3]{a}$ =(a) $^{1/3}$ (i.e cube root means power $\frac{1}{3}$)
:- in general $\sqrt[n]{a}$ = $a^{1/n}$

Also $\sqrt[n]{a^m}$ =$a^{m/n}$

TRICK TO BE USED BE A LAWYER



TYPE-1 STATE TRUE OF FALSE WITH REASONS

Q1. Is -2 a rational number?

Q2. Is every whole number a rational number?

Q3. Is every integer a whole number?

Q4. Is zero an integer?

Q5. Every rational number is a whole number?

TYPE-II:- FINDING RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS AND IRRATIONAL NUMBER BETWEEN TWO RATIONAL NUMBER.

Q6. Find three rational number between the following.

(a) $\frac{1}{2}$ and $\frac{1}{3}$ (b) -2 and 5 (c) 7 and 8

(d) 10 and 11 (e) 3 and 8 (f) $\frac{1}{2}$ and $\frac{1}{7}$

(g) 4 and 5 (h) $\frac{1}{2}$ and $\frac{1}{6}$ (i) 3 and 4

(j) $\frac{-3}{11}$ and $\frac{8}{11}$ (k) $\frac{-3}{13}$ and $\frac{9}{13}$ (l) $\frac{-2}{3}$ and $\frac{1}{4}$

(m) -2 and 6 (n) 3.2 and 3 (o) $\frac{1}{5}$and $\frac{1}{7}$

Q7. Find two rational numbers between the following irrational numbers.

(a) 2.1010 ---------------- and 2.50500500---------

(b) 1.30300300 --------- and   1.404004000 --------

(c) 0.89101001000 -------- and 0.9101001000--------

(d) 6.101001000 ----------- and 5.202002000 -------

(e) 0.23233233323333 ------- and 0.2121121111 ---------

(f) 0.5151151115 --------------- and 0.535335335 ---------

(g) 0.303003003 ------------------and 0.3010010001 --------

Q8. Find two irrational number between the following rational numbers.

(i) $\frac{1}{7}$ and $\frac{2}{7}$ (ii) $\frac{1}{3}$ and $\frac{1}{6}$ (iii) 0.12 and 0,13 (iv) $\frac{5}{7}$ and $\frac{9}{11}$

(v) 2 and 2.5 (vi) $\sqrt{2}$ and $\sqrt{3}$ (vii) 0.5 and 0.55 (viii) 0.1 and 0.12

TYPE-III:- BASED ON TERMINATING AND NON-TERMINATING NUMBERS.

Q9. Express the numbers in p/q form

(i) 0.15 (ii) 0.675 (iii) 0.00026 (iv) 15.75 (v) 8.0025 (vi) -25.876

Ans:- i) $\frac{3}{20}$ ii) $\frac{27}{40}$ iii) $\frac{13}{50000}$ iv) $\frac{63}{4}$ v) $\frac{3201}{400}$ vi) $\frac{-411}{16}$

Q10. Express each of the following in p/q form.

(i) $0.\overline{1}$ (ii) $0.\overline{2}$ (iii) $0.\overline{4}$ (iv) $0.\overline{5}$

Ans→ i) $\mathbf{\frac{1}{9}}$ ii) $\mathbf{\frac{2}{9}}$ iii) $\mathbf{\frac{4}{9}}$ iv) $\mathbf{\frac{5}{9}}$

TRICK TO CHECK YOUR ANSWER IN ABOVE
TYPE:- no of digits repeated in numerator
10-1 (if one digit is repeated)
For eg:- 0.1 ⇒ $\mathbf{\frac{1}{10-1}}$ **=** $\mathbf{\frac{1}{9}}$

Q11. Express each of following in p/q form

(i) $0.\overline{35}$ (ii) $0.\overline{585}$ (iii) $0.\overline{621}$ (iv) $0.\overline{37}$ (v) $0.\overline{75}$

Ans (i) $\mathbf{\frac{39}{99}}$ (ii) $\mathbf{\frac{585}{999}}$ (iii) $\frac{621}{999}$ (iv) $\frac{37}{99}$ (v) $\frac{75}{99}$

Q12. Express each of the following in p/q form.

(i) $5.\overline{2}$ (ii) $23.\overline{43}$ (iii) $0.\overline{32}$ (iv) $0.\overline{123}$

(v) $0.003\overline{52}$ (vi) $4.\overline{32}$ (vii) $15.\overline{712}$ (viii) $125.\overline{3}$

Ans i) $\mathbf{\frac{47}{9}}$ ii) $\frac{2320}{99}$ iii) $\frac{29}{90}$ iv) $\frac{111}{900}$ v) $\frac{349}{99000}$ vi) $\frac{389}{90}$ vii) $\frac{5185}{330}$ viii) $\frac{376}{3}$

TYPE-IV:- IDENTIFY AS RATIONAL OR IRRATIONAL

Q13.Determine if the following are irrational or rational.

(i) $\sqrt{7}$ (ii) $\sqrt{4}$ (iii) $2+\sqrt{3}$ (iv) $\sqrt{3}+\sqrt{5}$

(v) $(\sqrt{2}-2)^2$ (vi) $(2-\sqrt{2})(2+\sqrt{2})$ (vii) $\sqrt{5}-2$ (viii) $\sqrt{23}$

(ix) $\sqrt{225}$ (x) 0.3796 (xi) $3\sqrt{18}$ (xii) $\sqrt{1.44}$

(xiii) $\sqrt{\frac{9}{27}}$ (xiv) $\sqrt{64}$ (xv) $\sqrt{100}$ (xvii) $\sqrt{45}$

Q14. Find which variables x ,y, z etc. represent rational or irrational numbers.

i) $x^2 = 5$ (IRR) II) $y^2 = 9$ (R) III) $z^2 = 0.04$ (R)

IV) $u^2 = \frac{17}{4}$ (IRR) v) $v^2 = 3$ (IRR) vi) $w^2 = 27$ (IRR)

VII) $t^2 = 0.4$ (IRR)

Q15. Give example of two rational numbers whose.

i) Difference is a rational number Ans- ($\sqrt{2}$, $\sqrt{2}$)

ii) Difference is an irrational number Ans- ($4\sqrt{3}$, $2\sqrt{3}$

iii) Sum is a rational number Ans- ($\sqrt{5}$, -$\sqrt{5}$

iv) Sum is an irrational number Ans- ($2\sqrt{5}$, $3\sqrt{5}$)

v) Product is a rational number Ans- ($\sqrt{8}$, $\sqrt{2}$)

vi) Product is an irrational number Ans- ($\sqrt{2}$, $\sqrt{3}$)

vii) Quotient is a rational number Ans- ($\sqrt{8}$, $\sqrt{2}$)

viii) Quotient is an irrational number Ans- ($\sqrt{2}$, $\sqrt{3}$)

TYPE –V:- QUESTIONS BASED ON EXPONENTS

Q16. Evaluate each of the following

a) $\left(\frac{2}{11}\right)^4 \times \left(\frac{11}{3}\right)^2 \times \left(\frac{3}{2}\right)^3$ Ans- $\left(\frac{6}{121}\right)$

b) $\left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{3}{5}\right)^{-1}$ Ans- $\left(\frac{5}{486}\right)$

c) $2^{55} \times 2^{60} - 2^{97} \times 2^{18}$ Ans- (0)

d) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^2$ Ans- $\left(\frac{5}{2}\right)$

Q17. Simplify

a) $(3a^4 b^3)(18a^3 b^5)$ ans- $(54a^7 b^8)$

b) $\frac{3a^7b^6}{18a^6b^8}$ ans- $\left(\frac{1}{6}\,ab^{-2}\right)$

c) $\left(\frac{-2a^2}{6^3}\right)^3$ ans- $\left(\frac{-8a^6}{6^9}\right)$

d) $\frac{(4\times10^7)(6\times10^{-5})}{8\times10^4}$ ans- $\left(\frac{3}{100}\right)$

e) $\frac{4ab^2\,(-5ab^3)}{10a^2b^2}$ ans- $(-2b^3)$

Q18. Simplify each of the following

(i) $\frac{7^{n+2}-3\times7^{n+1}}{20\times7^n-2\times7^n}$

Since $a^n \times a^m = a^{m+n}$

$\Rightarrow \frac{7^n\times7^2-3\times7\times7^n}{20\times7^n-2\times7^n}$ (taking 7^n common from Nr and dr)

$\Rightarrow \frac{7^n(49-21)}{7^n(20-2)}$ $=\frac{28}{18}$ $\Rightarrow$ ans- $\frac{14}{9}$

ii) $\frac{5^{n+3}-6\times5^{n+1}}{9\times5^n-2^2\times5^n}$ $\Rightarrow$ (hint:- $\frac{5^n\times5^3-6\times5^n\times5}{9\times5^n-2^2\times5^n}$ 19)

iii) $\frac{16\times2^{n+1}-4\times2^n}{16\times2^{n+2}-2\times2^{n+2}}$ (same as above) (ans- $\frac{1}{2}$)

iv) $\frac{3^n\times9^{n+1}}{3^{n-1}\times9^{n-1}}$

v) $\frac{5\times25^{n+1}-25\times5^{2n}}{5\times5^{2n+3}-(25)^{n+1}}$

vi) $\frac{5^{n+3}-6\times5^{n+1}}{9\times5^n-2^2\times5^n}$

vii) $\frac{6\times(8)^{n+1}+16\times(2)^{3n-2}}{10\times(2)^{3n+1}-7\times(8)^n}$

Q19. If $\frac{a^n\times3^2\times3^n-(3^3)^n}{3^{3m}\times2^3}=\frac{1}{27}$, prove that m-n=1.

We have $\Rightarrow \frac{9^n\times3^2\times3^n-(3^3)^n}{3^{3m}\times2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{(3^2)^n\times3^2\times3^n-3^{3n}}{3^{3m}\times2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{2^n}\times3^2\times3^n-3^{3n}}{3^{3m}\times2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{2^n}\times3^2\times3^n-3^{3n}}{3^{3m}\times8} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n+2}-3^{3n}}{3^{3m}\times8} = \frac{1}{3^3}$ $\Rightarrow \frac{3^{3n}(3^2-1)}{3^{3m}(8)} = \frac{1}{3^3}$

$\Rightarrow \frac{3^{3n}\,(9-1)}{3^{3m}\times8} = \frac{1}{3^3}$ $\Rightarrow \frac{3^{3n}}{3^m} = \frac{1}{3^3}$

$\Rightarrow 3^{3n-3m} = 3^{-3}$

= on comparing

3n-3m = -3

= 3(n-3n) = -3

= n-m=-1 =

m-n=1

Hence proved

TYPE-VI :- PROVING G QUESTIONS BASED ON EXPONENTS

Q20. Assuming that x is a positive real numbers a, b and c are rational numbers show that.

i) $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$ =1

ii) $\left(\frac{x^a}{x^b}\right)^{1/ab} \times \left(\frac{x^b}{x^c}\right)^{1/bc} \times \left(\frac{x^c}{x^a}\right)^{1/ac}$ =1

iii) $\left(\frac{x^a}{x^b}\right)^{a^2+b^2+ab} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$ (use ch-2 identity)

iv) $\left(\frac{x^a}{x^b}\right)^{a+b} \left(\frac{x^b}{x^c}\right)^{b+c} \left(\frac{x^c}{x^a}\right)^{c+a} = 1$ (use identity)

v) $\left(\frac{x^a}{x^b}\right)^{a+b-c} \left(\frac{x^b}{x^c}\right)^{b+c-a} \left(\frac{x^c}{x^a}\right)^{c+a-b} = 1$

vi) $\left(\frac{x^a}{x^{-b}}\right)^{a^2+b^2-ab} \left(\frac{x^b}{x^{-c}}\right)^{b^2+c^2-bc} \left(\frac{x^c}{x^{-a}}\right)^{c^2+a^2-ca2(a^3+b^3+c^3)} =x$

vii) $\frac{x^{a(b-c)}}{x^{b\,(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c =1$

viii) $\frac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a x^b x^c)^4} =1$

Q21. Show that:-

i) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} =1$

Ans:- multiplying the numerator and denominator of three terms on LHS by x^a, x^b, x^c respectively, we obtain.

LHS= $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

$\Rightarrow \frac{x^a}{x^a+x^{b-a+a}+x^{c-a+a}} + \frac{x^b}{x^b+x^{a-b+b}+x^{c-b+b}} + \frac{x^c}{x^c+x^{b-c+c}+x^{a-c+c}}$

$\Rightarrow \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c}$

$\Rightarrow \frac{x^a+x^b+x^c}{x^a+x^b+x^c}$ =1 Ans

ii) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} =1$ (hint:- same as above)

iii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} =1$

Q22. Prove that:-

i) $\frac{a-1}{a^{-1}+b^{-1}} + \frac{a-1}{a^{-1}-b^{-1}} = \frac{2b^2}{b^2-a^2}$ (hint:- use laws of exponents.)

ii) $\frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}}$ =abc

iii) $(a^{-1} + b^{-1})^{-1} = \frac{ab}{a+b}$

iv) If a b c =1 then $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} =1$

$\Rightarrow \frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$

(Now since a b c- 1 :- $c=\frac{1}{ab}$) (also $\frac{1}{c} = ab$)

$\Rightarrow \frac{1}{\frac{b+ab+1}{b}} + \frac{1}{1+b+ab} + \frac{1}{\frac{1+1}{ab}+\frac{1}{a}}$

$\Rightarrow \frac{b}{b+ab+1} + \frac{1}{b+ab+1} + \frac{ab}{b+ab+1}$

$\Rightarrow \frac{b+ab+1}{b+ab+1}$ =1 hence proved

v) if abc =1 show that

$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$

TYPE:- VII:- FINDING THE VALUE OF X

Q23. Find the value of x if

i) $5^{x-3} \times 3^{2x-8} = 225$

$5^{x-3} \times 3^{2x-8} = 5^2 \times 3^2$

⇒ On comparing

x-3=2 → (x=5), 2x-8=2 → (x=5) :- x=5

ii) $2^{x-5} = 256$ (Ans- 13) (iii) $2^{x+3} = 4^{x-1}$ (Ans- x=5) (iv) $7^{2x+3}=1$ (Ans- $\frac{-3}{2}$)

v) $2^{5x+3}=8^{x+3}$ (Ans- x=3) (vi)$2^{3x-7} = 256$ (Ans- 5) (vii) $4^{2x} = \frac{1}{32}$ (Ans- x= $\frac{-5}{4}$

viii) 4x-1 × (0.5)3-2x = $\left(\frac{1}{8}\right)^x$ (Ans- x=$\frac{5}{7}$) (ix) $2^{3x-7}=256$ (Ans-x=5)

Q24. Solve the equations to find values of x.

i) $2^{2x+1} = 17.2^x - 2^3$

⇒ $2^{2x}.2 = 17.2^x - 2^3$

⇒ $2.(2^x)^2 = 17.2^x - 8$

⇒ $2.(2^x)^2 - 17.2^x + 8 = 0$

Put $2^x=y$

⇒ $2y^2-17y + 8=0$

$2y^2-16y - y+8=0$

2y(y-8) – (y-8) =0

(y-8) (2y-1) =0 = (y=8) or (y= ½)

But y=8 = $2^x = 8$ = $2^x=2^3$

:- x=3

Or y=$\frac{1}{2}$ ⇒ $2^x = \frac{1}{2} = 2^x = 2^{-1}$

:- (x=-1)

ii) $5^{2x+1} = 6.5^x - 1$ ($5^x=y$) (Ans- -1,0)

iii) $3^{2x+4} + 1 = 2.3^{x+2}$ ($3^x=y$) (Ans- -2)

iv) $2^{2x} - 2^{x+3} + 2^4 = 0$ (Ans-2)

v) 9^{x+2} 720+ 9^x (Ans- x=1)

TYPE- VIII:- FINDING VALUES ON PRIME FACTORISATION

Q25. If a, b, c are distinct positive prime integers such that a2 b3 c4 = 49392, find values of a,b,c .

Now:- $49392 = 2^4 x 3^2 x 7^3$

:- $a^2 b^3 c^4 = 49392$

$\Rightarrow a^2 b^3 c^4 = 2^4 \times 3^2 \times 7^3$

$\Rightarrow a^2 b^3 c^4 = 3^2 \times 7^3 \times 2^4$

$\Rightarrow$ a=3 , B=7 and C=4

2	49392
2	24696
2	12348
2	6174
3	3087
3	1029
7	343
7	49
7	7
	1

Q26. If $1176 = 2^a \times 3^b \times 7^c$ find a,b,c

(Ans- 3,1,2)

Q27. Given $4725 = 3^a 5^b 7^c$, find

i) integral values of a,b and c

(Ans:- a = 3,b=2, c=1)

ii) value of $2^{-a} 3^b 7^c$ (Ans:- $\frac{63}{8}$)

Q28. If $a = xy^{p-1}$, $b = xy^{9-1}$ and $c = xy^r$ prove that $a^{9-r} b^{r-p} c^{p-q} = 1$

(hint put a,b,c in LHS)

Q29. If $a = x^{m+n} y^l$, $b = x^{n+l} y^m$, $c = x^{l+m} y^n$ prove that $a^{m-n} b^{n-l} c^{l-m} = 1$

Q30. If $x = a^{m+n}$, $y = a^{n+l}$ and $z = a^{l+m}$, prove that $x^{m-} y^n z^l = x^n y^l z^m$ (hint:- solve LHS &RHS repeately0

TYPE –IX:- LAWS OF EXPONENTS

Q31. Simplify each of the following

i) $(625)^{-1/4}$ (Ans- 1/5)

ii) $\left(\frac{256}{81}\right)^{514}$ (Ans- $\frac{1024}{243}$)

iii) $\sqrt[5]{(32)^{-3}}$ (Ans- $\frac{1}{8}$)

iv) $\left[\left\{(625)^{\frac{-1}{2}}\right\}^{\frac{-1}{4}}\right]^2$ (Ans- 5)

v) $(256)-(4^{\frac{-3}{2}})$ (Ans- $\frac{1}{2}$)

vi) $\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{1}{(243)^{\frac{-1}{3}}}$

vii) $\sqrt{\frac{1}{4}} + (0.01)^{\frac{-1}{2}} - (27)^{2/3}$ (Ans- $\frac{3}{2}$

ix) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{5/4} \times (8)^{4/3}}$ (Ans- $\frac{3375}{512}$)

x) $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$ (Ans- 1)

xi) $(\sqrt{4})^{\frac{-3}{4}}$ (Ans- $\frac{1}{2^{3/4}}$)

xii) $(\sqrt{5})^{-3} \times (\sqrt{2})^{-3}$ $(Ans: \left(\frac{10^{1/2}}{100}\right)$

xiii) $(25)^{-1/3} \times \sqrt[3]{16}$ (Ans:- $\frac{2}{5} \times 10^{1/3}$)

xiv) $(\sqrt{4})^{-7} \times (\sqrt{2})^{-5}$ (Ans- $\frac{2^{1/2}}{2^{10}}$)

viii) $\left(\frac{1}{4}\right)^{-2}$ -3(8) $^{2/3}$ + $\left(\frac{9}{16}\right)^{\frac{-1}{2}}$ (Ans- $\frac{16}{3}$)

Q32. Simplify:-

i) $\sqrt{x^{-2}y^{3}}$ (Ans- $\frac{y^{3/2}}{x}$)

ii) $(x^{-2/3}y^{-1/2})^2$ (Ans- $\frac{1}{x^{4/3}y}$)

iii) $\sqrt[4]{\sqrt[3]{x^2}}$ (Ans- $x^{1/6}$)

iv) $\sqrt[3]{xy^2} \div x^2y$ $(x^{-5/3}y^{-1/3})$

v) $(\sqrt{x})^{-2/3}\sqrt{y^4} \div \sqrt{xy^{-1/2}}$ (Ans- $\frac{y^{9/4}}{x^{5/6}}$)

Q33. If x, y, z are positive real numbers.

Show that $\sqrt{x^{-1}y} \times \sqrt{y^{-1}z} \times \sqrt{z^{-1}x}$ =1

Q34. If $\left(\frac{x^{-1}y^2}{x^3y^{-2}}\right)^{1/3} \div \left(\frac{x^6y^{-3}}{x^{-2}y^2}\right)^{1/2}$ = x^a y^b prove that a+b=-1, where x and y are different positive primes.

(hint:- find value of a & b i.e a = $\frac{-16}{3}$, b = $\frac{13}{3}$) then add and show.

Q35. Find value of x

i) if 25^{x-1} = 5^{2x-1}-100

(hint:- 5^{2x-2}- 5^{2x-1} =-100 , $\Rightarrow$ $5^{2x-1}(5^{-1}-1)$ = -100) (Ans- x=2)

ii) $\sqrt{\left(\frac{3}{5}\right)^{1-2x}}$ =4 $\frac{17}{27}$ (Ans- $\frac{7}{2}$)

iii) $\sqrt[3]{\left(5^0+\frac{2}{3}\right)^2}$ = $(0.6)^{3-2x}$ (Ans- x=$\frac{11}{6}$)

iv) $2^3(5^0+3^{2x})$ = 8 $\frac{8}{27}$ (Ans- x= -3/2)

v) $3(2^x+1) - 2^{x+2} + 5 = 0$ (Ans- x=3)

vi) $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$ (Ans- 1)

vii) $(2^3)^4 = (2^2)^x$ (Ans-6)

viii) $5^{x-2} \times 3^{2x-3}$ = 135 (Ans- 3)

ix) $2^{x-7} \times 5^{x-4}$ = 1250 (Ans- 8)

x) $(\sqrt[3]{4})^{2x+1/2} = \frac{1}{32}$ (Ans- -4)

xi) $\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$ (Ans- -7)

Q36. Solve the equations.

i) $3^{x+1} = 27\times 3^4$ (Ans- 6)

ii) $4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$ (Ans- $\frac{5}{7}$)

iii) $\sqrt{\frac{a}{b}} = (b/a)^{1-2x}$ (Ans- $\frac{3}{4}$)

iv) $27^x = \frac{9}{3^x}$ (Ans- $\frac{1}{2}$)

Q37. Prove that

(i) $\left[\left\{\frac{x^{a(a-b)}}{x^{a(a+b)}}\right\} \div \left\{\frac{x^{b(b-a)}}{x^{b(b+a)}}\right\}\right]^{a+b}$ =1

(ii) $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-a}}$ =1

(iii) $\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b} \left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c} \left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c}$ =x 2($a^2 + b^3 + c^3$)

(hint:- use identity $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$)

(iv) $\left\{\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right\}^{\frac{a}{a+1}} = x$

(v) $\left(\frac{a^{x+1}}{a^{y+1}}\right)^{x+y} \left(\frac{a^{y+2}}{a^{z+2}}\right)^{y+z} \left(\frac{a^{z+3}}{a^{x+3}}\right)^{3+x}$ =1

(vi) $\left(\frac{3^a}{3^b}\right)^{a+b} \left(\frac{3^b}{3^c}\right)^{b+c} \left(\frac{3^c}{3^a}\right)^{c+a}$ =1

(vii) $\frac{\left(a+\frac{1}{b}\right)^m \times \left(a-\frac{1}{b}\right)^n}{\left(b+\frac{1}{a}\right)^m \times \left(b-\frac{1}{a}\right)^n} = \left(\frac{a}{b}\right)^{m+n}$

(viii) $\sqrt[lm]{\frac{x^l}{x^m}} \times \sqrt[mn]{\frac{x^m}{x^n}} \times \sqrt[nl]{\frac{x^n}{x^l}}$ =1

(ix) $\left(\frac{x^{a+b}}{x^c}\right)^{a-b} \left(\frac{x^{b+c}}{x^a}\right)^{b-c} \left(\frac{x^{c+a}}{a^b}\right)^{c-a}$ =1

(x) $\{(x^a)^b\}^{\frac{1}{ab}} \left\{\left(x^b\right)^c\right\}^{\frac{1}{bc}} \{(x^c)^a\}^{\frac{1}{ac}}$ =1

TYPE-X:- PROVING QUESTIONS (HOTS)

Q38. If $a^x = b$, $b^y = c$ and $c^3 = a$ prove that xyz =1

We know that $a^{xyz} = (a^x)^{yz}$ [:- $a^x = b$]

$a^{xyz} = (b)^{yz}$

$a^{xyz} = (b^y)^z$

$a^{xyz} = a$

:- on comparing xyz =1

Q39. If $a^x = b^y c^z$ and $b^2 = ac$, prove that

$Y = \frac{2xz}{x+z}$

Let – $a^x = b^y = c^z$ =k,then

$A = k^{1/x}$, $b = k^{1/y}$, $c = k^{1/z}$

Now $b^2 = ac$

$\Rightarrow \left(k^{1/y}\right)^2 = k^{1/x} \times k^{1/2}$

$$k^{2/y} = k^{\frac{1}{x}+\frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z} = \frac{2}{y} = \frac{x+z}{xz}$$

$$\Rightarrow y = \frac{2xz}{x+z}$$

Q40. (i) if 2x = 3y = 12z show that $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$,

(ii) if 3x = 5y $(75)^3$, show that $z = \frac{xy}{2x+y}$

(iii) $x = 2^{1/3} + 2^{2/3}$ show that $x^3 - 6x = 6$

Q41. Simplify or prove that

(i) $\left(\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}}\ \sqrt{5}\right) \times \sqrt[6]{3 \times 5^{6}} = \frac{3}{5}$

(ii) $9^{3/2} - 3 \times 5^{0} - \left(\frac{1}{81}\right)^{-1/2} = 15$

(iii) $\left(\frac{1}{4}\right)^{-2} - 3 \times 8^{2/3} \times 4^{0} + \left(\frac{9}{16}\right)^{-1/2} = \frac{16}{3}$

(iv) $\frac{2^{1/2} \times 3^{1/3} \times 4^{1/4}}{10^{-1/5} \times 5^{3/5}} \div \frac{3^{4/3} \times 5^{-7/5}}{4^{-3/5} \times 6} = 10$

(iv) $\frac{2^{1/2} \times 3^{1/3} \times 4^{1/4}}{10^{-1/5} \times 5^{3/5}} \div \frac{3^{4/3} \times 5^{-7/5}}{4^{-3/5} \times 6} = 10$

(v) $\sqrt{\frac{1}{4}} + (0.01)^{-1/2} - (27)^{2/3} = \frac{3}{2}$

(vi) $\frac{2^{n} + 2^{n-1}}{2^{n+1} - 2^{n}} = \frac{3}{2}$

(vii) $\left(\frac{64}{125}^{-2/3} + \frac{1}{\left(\frac{256}{625}\right)^{1/4}}\right) + \left(\sqrt{\frac{25}{\sqrt[3]{64}}}\right) = \frac{61}{16}$

(viii) $\frac{3^{-3} \times 6^{2} \times \sqrt{98}}{5^{2} \times \sqrt[3]{\frac{1}{25}} \times (15) \times 3^{1/3}} = \sqrt[28]{2}$

TYPE- XI:- LAWS OF RATIONALISATION

Q42. Simplify :-

(i) $\sqrt[5]{16} \times \sqrt[5]{2}$ (Ans- 2) (ii) $\frac{\sqrt[4]{243}}{\sqrt[4]{3}}$ (Ans- 3)

(iii) $\sqrt[3]{4} \times \sqrt[3]{16}$ (Ans- 4) (iv) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$ (Ans-5)

(v) $(3+2\sqrt{2})(3-2\sqrt{2})$ (Ans-1) (vi) $(\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$ (Ans-6)

(vii) $(2\sqrt{5} + 3\sqrt{2})^2$

Q43. Rationalise the denominator

(i) $\frac{2}{\sqrt{3}}$ (Ans- $\frac{2\sqrt{3}}{3}$) (vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$ (Ans:-$\frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$

(ii) $\frac{1}{3+\sqrt{2}}$ (Ans-$\frac{3-\sqrt{2}}{7}$) (vii) $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$ (Ans:- $\frac{47+21\sqrt{5}}{2}$)

(iii) $\frac{5}{\sqrt{3}-\sqrt{5}}$ (Ans:- $\frac{5}{2}(\sqrt{3}+\sqrt{5})$)

(iv) $\frac{1}{7+3\sqrt{2}}$ (Ans- $\frac{7-3\sqrt{2}}{31}$)

(v) $\frac{2\sqrt{7}}{\sqrt{11}}$ (Ans- $\frac{2}{11}\sqrt{77}$)

(viii) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ (Ans:- $\frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$)

(ix) $\frac{b^2}{\sqrt{a^2+b^2}+a}$ (Ans:- $\sqrt{a^2+b^2}-a$)

Q44. Find value to 3 decimal reces given that

$\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{10}=3,162$ and $\sqrt{5}$ =2.236

(i) $\frac{\sqrt{2}+1}{\sqrt{5}}$ (Ans- 1.079) (ii) $\frac{2-\sqrt{3}}{\sqrt{3}}$ (Ans- 0.154)

(iii) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$ (Ans- 0.655) (iv) $\frac{2+\sqrt{3}}{3}$ (Ans- 1.2444)

(v) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$ (Ans- 4.975) (vi) $\frac{\sqrt{5}+1}{\sqrt{2}}$ (Ans- 2.288)

(vii) $\frac{2+\sqrt{3}}{3}$

Q45. If a and b are rational numbers, find the values of a and b in each of following .

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1}=$ a+b$\sqrt{3}$

$\Rightarrow$ ***Rationalising***

$\Rightarrow \frac{(\sqrt{3}-1)(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1}=\frac{(\sqrt{3})^2+1-2\sqrt{3}}{(\sqrt{3})^2-1}$

$\Rightarrow \frac{3+1-2\sqrt{3}}{3-1}=\frac{4-2\sqrt{3}}{2} \Rightarrow 2-\sqrt{3}$

Since $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ =a+b$\sqrt{3}$

:- 2-$\sqrt{3}$ = a+b$\sqrt{3}$

On compaling a=2, b=-1

(ii) $\frac{3+\sqrt{7}}{3-\sqrt{7}}=\frac{a+b\sqrt{7}}{(a=8,b=3)}$

(iii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=\frac{a+b\sqrt{3}}{(a=11,b=-6}$

(iv) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{a+b\sqrt{15}}{(a=4,b=1)}$

(v) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{a-b\sqrt{6}}{(a=2,b=-\frac{5}{6})}$

(vi) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=\frac{a+b\sqrt{3}}{(a=-1,b=1)}$

(vii) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}}=$ a+b$\sqrt{5}$ (a=$\frac{-61}{29}$ b= $\frac{-24}{29}$

Q46. Simplify each of the following

(i) $\frac{3}{5-\sqrt{3}}+\frac{2}{5+\sqrt{3}}$ (Ans- $\frac{25+\sqrt{3}}{22}$)

(ii) $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$ (Ans- $\frac{42}{11}$)

(iii) $\frac{\sqrt{5}-2}{\sqrt{5}+2}$ - $\frac{\sqrt{5}-2}{\sqrt{5}-2}$ (Ans:- - $8\sqrt{5}$)

(iv) $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5$

(v) $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}=2$

(vi) $\frac{1}{2+\sqrt{3}}+\frac{1}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$ (Ans:- 0)

(vii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$ (Ans:- 0)

Q47. Evaluate $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$, is being given that $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$ (Ans:- 5.398)

(hint:- $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$\Rightarrow \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$

$\Rightarrow \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$

$\Rightarrow 3\sqrt{10}-3\sqrt{5}$

$\Rightarrow 3\sqrt{10}-\sqrt{5}$)

Q48. Simplify , $\sqrt{2}=1.4142$, $\sqrt{3}=1.732$, $\sqrt{5}=2.2360$, $\sqrt{6}=2.4495$ and $\sqrt{10}=3.162$

(i) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ (Ans:- 0.02

(ii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ (Ans:- 14.0710)

(iii) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$ (Ans:- 11)

(iv) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}$ - $\frac{7-3\sqrt{5}}{3-\sqrt{5}}$ (Ans:- $\sqrt{5}$)

TYPE- XII:- TO FIND VALUE GIVEN EXPRESSIONS

Q49. If $x=2+\sqrt{3}$, find value of $x^2+\frac{1}{x^2}$

$\Rightarrow$ We know that $(a+b)^2 = a^2+b^2+2ab$

Put a=x , $b=\frac{1}{x}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$\Rightarrow x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

Let $x=2+\sqrt{3}$

Now $\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{y-3}$, $2-\sqrt{3}$

Now $x^2+\frac{1}{x^2}=(2+\sqrt{3}+2-\sqrt{3})^2$-2

$X^2+\frac{1}{x^2} = (4)^2 - 2 = 16-2=14$

Q50. If x=3-2$\sqrt{2}$ find x2 +$\frac{1}{x^2}$ (Ans:- 34)

Q51. If x=1-$\sqrt{2}$, find value of $\left(x-\frac{1}{x}\right)^3$ (Ans:- 8)

Q52. If x= $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and y= $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find x2 +y2. (Ans:- 98)

Q53. If a = $\frac{2-\sqrt{5}}{2+\sqrt{5}}$ and b = $\frac{2+\sqrt{5}}{2-\sqrt{5}}$, find a2-b2 (Ans:- -144$\sqrt{5}$)

Q54. If x =2+$\sqrt{3}$, find value of $x^3+\frac{1}{x^3}$ (Ans:- 52)

Q55. If x= 3+$\sqrt{8}$, find value of $x^2+\frac{1}{x^2}$ (Ans:- 34)

Q56. If x= $\frac{1}{2-\sqrt{3}}$, find $x^3 - 2x^2 - 7x+5$ (Ans:-3)

Q57. Simplify:-

(i) $\sqrt{3+2\sqrt{2}}$

$\Rightarrow \sqrt{2+1+2\sqrt{2}} \quad = \sqrt{(\sqrt{2})^2+(1)^2+2\times\sqrt{2}\times 1}$

$(a+b)^2 = a^2+b^2+2ab = \sqrt{(\sqrt{2}+1)^2}$

$\sqrt{(\sqrt{2}+1)^2} = ((\sqrt{2}+1)^2)$ = (Ans:- $\sqrt{2}+1$)

(ii) $\sqrt{3-2\sqrt{2}}$ (Ans:- $\sqrt{2}-1$)

(iii) $\sqrt{5+2\sqrt{6}}$ (Ans:- $\sqrt{3}+\sqrt{2}$)

Q58. Find value of

(i) if x= $\sqrt{2}+1$,then x-$\frac{1}{x}$ (Ans:- 2)

(ii) if x = 3+ 2$\sqrt{2}$, find $\sqrt{x}-\frac{1}{\sqrt{x}}$, (Ans:- 2)

(iii) if x = $\frac{2}{3+\sqrt{7}}$, then$(x-3)^2$ (Ans:- 7)

(iv) if x + $\sqrt{15}$ = 4, find x+ $\frac{1}{x}$ (Ans:- 8)

(v) if x = $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and y= $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then x+y+xy= ______? (Ans:- 9)

(vi) if x= $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, y=$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$,then x^2+ xy +y^2________? (Ans:- 99)

(vii) if x=$\sqrt[3]{2+\sqrt{3}}$ find $x^3+\frac{1}{x^3}$ (Ans:- 4)

(viii) if x = $\sqrt{6}+\sqrt{5}$, x^2 +1________2? (Ans:- 20)

Q59. Short questions

(i) positive square root of $7+\sqrt{48}$ ________? (Ans:- $2+\sqrt{3}$)

(ii) if $\sqrt{2}$ =1,4142, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is _______? (Ans:- 0.4142)

(iii) if $\sqrt{13-a\sqrt{10}}=\sqrt{8}+\sqrt{5}$, find a _______? (Ans:- 4)

(iv) rationalization factor of $\sqrt{5}-2$. (Ans:- $2+\sqrt{5}$)

BOOSTER QUESTIONS (SELF PRACTICE)

Q60. If x=$\frac{1}{2-\sqrt{3}}$ find value of $\frac{1}{x}$ and x-1?

Q61. If x=1-$\sqrt{2}$ find value of x^2- $\frac{1}{x^2}$?

Q62. If x = $\frac{3+\sqrt{5}}{2}$, find value of x^2 +$\frac{1}{x^2}$?

Q63. If x= $3\sqrt{5}$ + $2\sqrt{2}$ and y= $3\sqrt{5}$ - $2\sqrt{2}$ find value of.

(i) x+y (ii) x-y (iii) x^2 +y^2

Q64. If x= $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and y= = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ find value of

(i) (x+y)2 (ii) (x-y)2 (iii) x^2y +xy^2

Q65. Solve for x

(i) $2^{5x} \div 2^x = \sqrt[4]{16}$ (ii) $(343)^{\frac{2}{x}}$ = 49

(iii) $(-5)^{x-2} = \left(\frac{1}{8}\right)^3$ (iv) $(16)^{2x-1} = 4^{x-5}$

Q66. Simplify $\sqrt{2+\sqrt{3}}$ + $\sqrt{2-\sqrt{3}}$, when it is given that $\sqrt{6}$ =2.499 (Ans:- 2.499)

Q67. If x= $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ + $\frac{2-\sqrt{3}}{2+\sqrt{3}}$ + $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ find value of x2 + $\left(\frac{53}{x}\right)^2$.

Q68. Prove that if a=1+ $\sqrt{2}$ +$\sqrt{3}$ and b= 1+ $\sqrt{2}$ -$\sqrt{3}$ prove that a^2 +b^2- 2a -2b =8

Q69. If x=$\frac{1}{2-\sqrt{3}}$, find value of x^3-2^{x2}-7x+5.

Q70. Find value of $\left\{(23+2^2)^{\frac{2}{3}}+(130-19)^{\frac{1}{2}}\right\}^2$.

Q71. Find value of x if $\frac{3^{5x}\times(81)^2\times 656)}{3^{2x}}$ =3^7. Find value of x.

Q72. If $\frac{x}{x^{-5}}$= $8x^{-1}$, find x.

Q73. If abc =1, show that:

$$\left(1+a+\frac{1}{b}\right)^{-1}+\left(1+b+\frac{1}{c}\right)^{-1}+\left(1+c+\frac{1}{a}\right)^{-1}=1$$

Q74. If x=$\frac{4\sqrt{3}}{2-\sqrt{2}}$ -$\frac{30}{4\sqrt{3}-3\sqrt{2}}$ - $\frac{3\sqrt{2}}{3+2\sqrt{3}}$ find value of (x4 + x2 + 3) + $\frac{1}{(x^4+x^2+3)}$.

Q75. If x=$\sqrt[3]{28}$ and y = $\sqrt[3]{27}$ find value of x+y -$\frac{1}{x^2+xy+y^2}$? (Ans:- 6)

Q76. If x=$\frac{1}{2-\sqrt{3}}$, evaluate x^2-4x+9

Hence evaluate x3-4x2 +9x+10. (Ans:- 8,26+8$\sqrt{3}$)

Q77. If $\frac{9^{m+1}(3^{\frac{-n}{2}})^{-2}-(27)^n}{(3^m\times 2)^3}=\frac{1}{729}$ prove that m-n=2

Q78. If x= $\frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, prove that bx^2-ax + b=0.

Q79. If x=3 , find value of $\left(x^{\frac{1}{3}+} + x^{\frac{-1}{3}}\right)\left(x^{2/3} + x^{-2/3} - 1\right)$.